Down to Earth
KS3

Teacher Guide

Impactor Speed

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Impactor Speed

The aim of this activity is to use vectors and Pythagoras' theorem to work out the speed at which the impactor hits the comet.

Objectives

The students will:

- Remember that velocity is a vector
- Calculate the relative velocity of the comet and impactor

Resources required

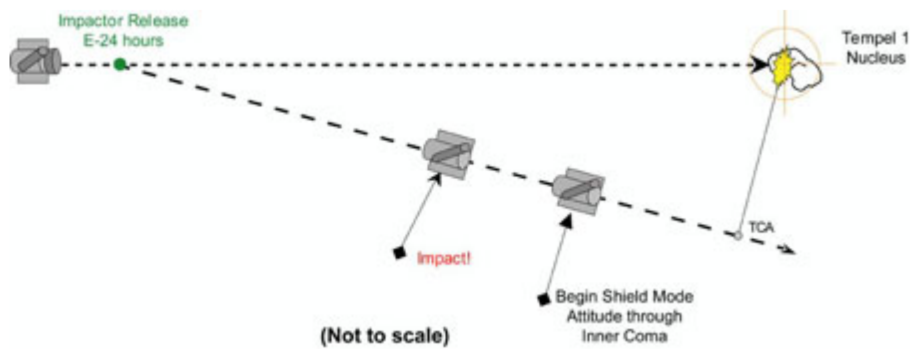
Paper, pencils, example sheet.

Teaching activities

Introduce the Deep Impact mission using the fact sheet. The main point is that a crater will be made in the side of comet Tempel 1 to help us understand what a comet is made of.

The question the students must answer is:

1. **How fast will the impactor be moving when it hits Tempel 1?**



The x, y, and z velocities (in kilometers per second) for the comet

x = 27.09 km/s
y = -11.40 km/s
z = -5.46 km/s

and the impactor spacecraft

x = 18.41 km/s
y = -12.67 km/s
z = -0.16 km/s

a) From these, can you determine the velocity of the impactor spacecraft relative to the comet at the time of impact?

Discussion

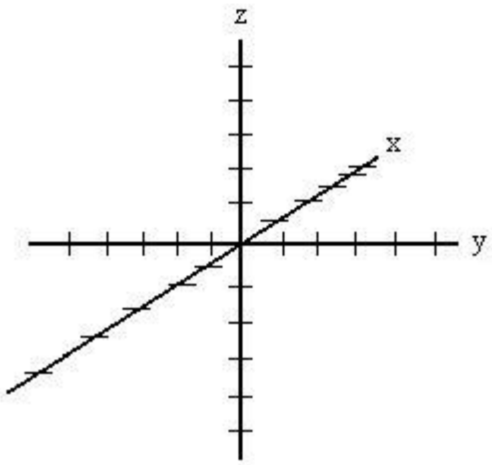
The main point that the students need to understand is the relative velocity of the comet and impacting spacecraft needs to be determined.

Scientists need to know how fast the impactor spacecraft will be traveling relative to the comet in order to calculate the energy of collision. This velocity can then in turn be used to try and work out how big the crater might be.

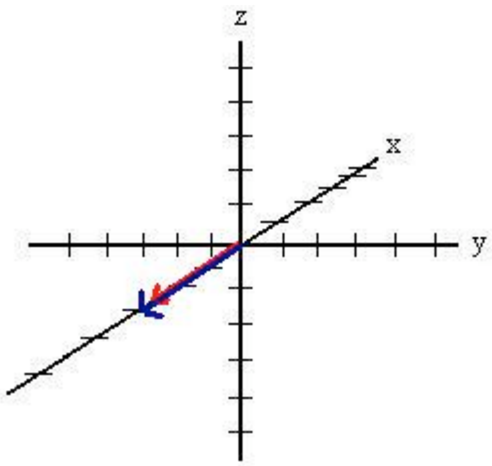
As an example of why the relative velocity is needed, consider two cars involved in a rear-end collision. If both cars are moving at the time they collide, it isn't the velocity of the first car or the second car that would be used to calculate the energy of impact, it's the difference between them. So a car traveling at 60 mph striking a car traveling 55 mph transfers the same amount of energy at collision as a car traveling 5 mph striking a stationary car.

Another complication that has to be taken into account is the fact that the comet and impactor spacecraft will be moving through three-dimensional space, instead of the "one dimensional" space of the car example. Fortunately this can be fixed by thinking of the motion one piece at a time, using vectors.

If you assign the same x-y-z set of axes for both objects, their movement can be broken down into three components, an x-component, a y-component and a z-component. This gives you the velocity that the object is moving along that axis. The overall velocity of the object can be determined by simply adding up the vectors. Since the three vectors will be at right angles to each other (since they're along the three right-angle axes), we can just use the three dimensional form of the Pythagorean theorem:



where $v^2 = x^2 + y^2 + z^2$



Looking back at the car example, let's say that the cars were moving along the x-axis. This means that their velocities (in mph) along the three axes would be:

Car 1: (60, 0, 0)
Car 2: (55, 0, 0)

This means that the overall velocities of the cars would be:

Car 1: $v = \left|\sqrt{x^2 + y^2 + z^2}\right| = \left|\sqrt{(60)^2 + (0)^2 + (0)^2}\right|$ = 60 mph

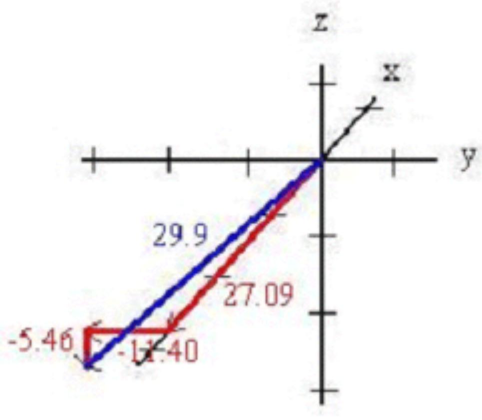
Car 2: $v = \left|\sqrt{x^2 + y^2 + z^2}\right| = \left|\sqrt{(55)^2 + (0)^2 + (0)^2}\right|$ = 55 mph

The last piece of information the students need is the x, y, and z velocities (in kilometers per second) for the comet (27.09, -11.40, -5.46) and the impactor spacecraft (18.41, -12.67, -0.16). From these, can the students determine the velocity of the impactor spacecraft relative to the comet at the time of impact?

Answers:

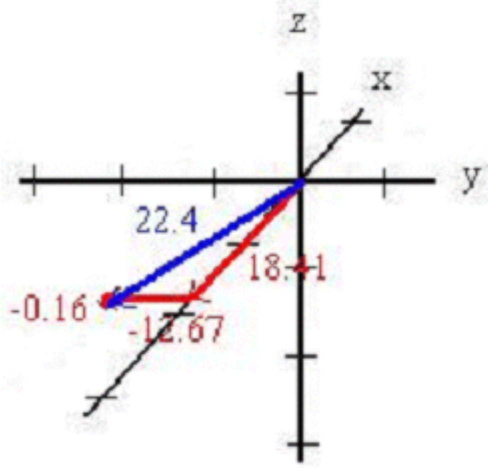
Looking at the Deep Impact mission, the x, y, and z velocities (in kilometers per second) for the comet Tempel 1 at the time of impact are (27.09, -11.40, -5.46). This means the overall velocity of the comet is:

$$
\begin{aligned}
v &= \left|\sqrt{x^2 + y^2 + z^2}\right| \\
&= \left|\sqrt{(27.09)^2 + (-11.40)^2 + (-5.46)^2}\right| \\
&= 29.9 \ \mathrm{km/s}
\end{aligned}
$$



And the x, y, and z velocities (in kilometers per second) for the impactor spacecraft at the time of impact are (18.41, -12.67, -0.16). This means the overall velocity of the impactor spacecraft is:

$$
\begin{aligned}
v &= \left|\sqrt{x^2 + y^2 + z^2}\right| \\
&= \left|\sqrt{(18.41)^2 + (-12.67)^2 + (-0.16)^2}\right| \\
&= 22.4 \ \mathrm{km/s}
\end{aligned}
$$



Now, you might be tempted at this point to say that the comet is just 7.5 km/s (29.9-22.4) faster than the impactor spacecraft, so the relative velocity of the impactor spacecraft will be 7.5 km/s. Unfortunately, this does not take into account the fact that the two objects will not be heading in exactly the same direction, like the cars were in the earlier example.

So, instead, to determine the *relative* velocity of the impactor spacecraft, you have to subtract each component (x, y, and z) of the comet from the impactor spacecraft, and then run the same type of 3D Pythagorean theorem calculation on the resulting relative components.

This works even in the "one dimensional" car example. The relative velocities of the second car will be: (55-60, 0-0, 0-0) or (-5, 0, 0). This means the overall relative velocity of the second car to the first is:

$$v = \left|\sqrt{x^2 + y^2 + z^2}\right| = \left|\sqrt{(-5)^2 + (0)^2 + (0)^2}\right| = 5 \text{ mph}$$

Again looking at Deep Impact, the relative velocities of the impactor spacecraft then will be: (18.41-27.09, -12.67-(-11.40), -0.16-(-5.46)) or (-8.68, -1.27, 5.30). This means the overall relative velocity of the impactor spacecraft is:

$$v = \left|\sqrt{x^2 + y^2 + z^2}\right| = \left|\sqrt{(-8.68)^2 + (-1.27)^2 + (5.30)^2}\right| = 10.2 \text{ km/s}$$

So, the impactor spacecraft will transfer the same amount of energy to the comet as it would if the comet were stationary and the impactor spacecraft were moving at 10.2 km/s.

See the follow on activity: Collision energy